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LETTER TO THE EDITOR

Test of universality for Ising-correlated site percolation

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Abstract. A universal amplitude ratio was determined from cluster numbers in the Ising model at twice the critical temperature. Our Monte Carlo data agree with those for random percolation (infinite temperature), in agreement with expectations from renormalisation group arguments, but in disagreement with earlier Monte Carlo simulations of Stoll and Domb on much smaller systems.

Universality of critical amplitudes at second-order phase transitions (Betts *et al* 1971) connects various quantities in different models or materials. Roughly speaking, if we form a dimensionless combination of quantities (each of which is not dimensionless), and if this combination approaches a finite non-zero limit at the critical point (though each of the original quantities diverges or vanishes there), then usually this combination is the same for all different materials or models which fall within the same universality class. For example, one may look at the ratio of susceptibilities at a small distance from the critical temperature T_c, with one susceptibility measured above T_c and the other below T_c, at the same distance from T_c. Each of these susceptibilities diverges at the critical point, but their ratio R approaches a finite limit according to scaling theory. Universality then says that this ratio, R, approaches the same limit for all similar magnets, for example for all ferromagnetic Ising models in three dimensions.

For percolation (see Stauffer 1979, Essam 1980 or Adler *et al* 1982 for reviews) an analogous universality statement (Marro 1976) asserts that the ratio of the second moment of the cluster size distribution (often called the ratio of mean cluster sizes) approaches the same limit for different lattices of the same dimensionality, provided the two second moments are taken at the same distance from the percolation threshold p_c. A theoretical foundation for universality in percolation theory was given by Aharony (1980). Somewhat related is the ratio of the number of s-clusters at p_c to the number of s-clusters at that concentration $p_{max}(s)$ where the number of s-clusters has a maximum as a function of concentration p, at fixed s. We call the first ratio the 'susceptibility ratio' and the second one the 'cluster number ratio'.

Both ratios have become of practical importance recently: Herrmann *et al* (1982) found from the susceptibility ratio that a kinetic gelation process does not belong in the same universality class as random percolation, even though the critical exponents were indistinguishable. Also, Djordjevic *et al* (1982) used the assumed universality

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of the cluster number ratio to determine very accurately the percolation threshold in the square and honeycomb lattices. On the other hand, Gawlinski and Stanley (1981) found deviations in the susceptibility ratio when they compared their results for continuum percolation with those of lattice percolation (Hoshen *et al* 1979). Thus a more careful investigation of these ratios is appropriate.

Our work was motivated particularly by the earlier study of Stoll and Domb (1979) who looked, among many other properties, at the cluster number ratio in random percolation on the square lattice, as well as for interacting percolation in the Ising ferromagnet at $T = 2T_c$. At infinite temperature (random percolation) they found a cluster number ratio of 5.4, whereas at $T = 2T_c$ this ratio was only 4.0. (Series expansions give 4.95 ± 0.15 (Djordjevic *et al* 1982), and Monte Carlo simulations of $35\,000 \times 35\,000$ triangular lattices give 5.0 ± 0.1 (Margolina 1982).) Thus universality seems not to hold in this case. On the other hand, renormalisation group arguments (Klein *et al* 1978; for a review see Stauffer *et al* 1982) indicate that all Ising-correlated percolation models should fall into the same universality class as random percolation, as long as the magnetic correlation length remains finite (i.e. away from the Curie point). The Monte Carlo work of Stoll and Domb (1979) was based on rather small lattices (110×110), and these authors had already pointed out that this system size may not have been sufficient. For two-dimensional percolation the boundary effects are particularly important since the connectivity exponent $\nu = \frac{4}{3}$ is unusually large, causing the connectivity length to diverge more strongly than the magnetic correlation length in Ising models ($\nu = 1$). Suspecting that finite-size effects were responsible for the deviation in the cluster number ratio, we therefore repeated the experiment of Stoll and Domb with much larger lattices.

Standard Monte Carlo techniques (multi-spin coding followed by multi-label cluster characterisation; see Stauffer (1982) for a review) were used to count clusters in thermal equilibrium of an Ising square lattice at $T = 2T_c$, for 400×400 and 800×800 lattices. Periodic boundary conditions were used for the magnetic interactions but not for the cluster counts (Herrmann and Stauffer 1980).

Following Stoll and Domb, we plotted the fraction of spins in the infinite cluster, and the second moment of the cluster size distribution $\Sigma\, s^2 n_s$, as a function of concentration ($p = \frac{1}{2} + \frac{1}{2} \times$ magnetisation) by varying the magnetic field in the kinetic Glauber model. Since the critical exponents β and γ are believed to be known exactly ($\frac{5}{36}$ and $\frac{43}{18}$, respectively), we plotted the strength of the infinite cluster and the mean cluster size raised to the power $1/\beta$ and $-1/\gamma$, respectively. Then we determined the effective critical points by extrapolating these quantities to zero visually. We found p_c to be 0.562 and 0.555 for the 400×400 and 800×800 lattice, respectively. With the same method Stoll and Domb had $p_c = 0.569$ for their 110×110 lattice. Extrapolating these estimates to infinite system size, we find

$$p_c(T/T_c) = 0.55$$

corresponding to a magnetic field $\mu H/k_B T = 0.033$.

A direct determination of the cluster number ratio indicates that this ratio decreases systematically with increasing cluster size s and that its asymptotic limit seems to be below 5, as found by Stoll and Domb. However, we regard this effect as spurious and due to the finite size even for our systems. For random percolation, the numbers of large clusters simulated in a finite lattice right at $p = p_c$ are known to be enhanced by boundary effects (Hoshen *et al* 1979, Margolina 1982 as cited in Stauffer 1982), even for 4000×4000 and $35\,000 \times 35\,000$ lattices. We reproduced this effect by a simple 800×800 simulation of random triangular site percolation: data at $p = p_c$ deviate from the asymptotic behaviour for cluster sizes larger than 10^2. For Ising-correlated percolation, figure 1 shows clearly the finite-size effects at $p = p_c$ (lower part of the figure). Asymptotically one expects (Stauffer 1979, Essam 1980) for the number (per lattice site) of s-clusters: $n_s \propto s^{-\tau}$ with $\tau = 2 + 1/\delta = 2 + \beta/(\beta + \gamma) = \frac{187}{91}$. For our larger system size, the data of figure 1 exhibit a plateau in $s^\tau n_s$ versus s; and we take this plateau value $s^\tau n_s = 0.032 \pm 0.001$ as the asymptotic limit at p_c.

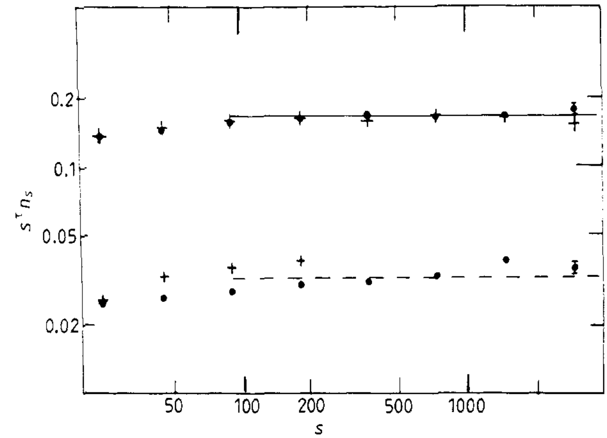

Figure 1. Log–log plot of the scaled cluster numbers versus cluster size. The upper part refers to data at p_{max} where the cluster numbers n_s have a maximum as a function of p; the lower part refers to the effective percolation threshold $p_c = 0.555$ or $\mu H/k_B T = 0.034$. The straight lines indicate the asymptotic value determined from this plot; their distance corresponds to the cluster number ratio 5.1. The dots are for 800×800 lattices, with the two largest statistical error bars shown, the crosses refer to 400×400 (less precision). One sees that finite-size effects are more disturbing at p_c than at p_{max}.

For the cluster numbers at their maximum below p_c, this finite-size effect is seen from figure 1 (upper part) to be much lower; we extrapolate $s^\tau n_s(p_{max}(s))$ to 0.1635 ± 0.002. Then the ratio between these two extrapolations gives the cluster number ratio as 5.1 ± 0.2, compatible with the ratio 5.0 in random percolation and appreciably higher than the estimate 4.0 of Stoll and Domb.

For the susceptibility ratio, figure 2 shows for the effective $p_c = 0.555$ of the 800×800 system the variation of the second moments $\Sigma s^2 n_s$ (excluding the largest cluster) with distance from the percolation threshold. The differences between the 400×400 data and the 800×800 data indicate that only concentrations rather far away from the percolation threshold should be relied upon. These data agree surprisingly well with the expected susceptibility exponent $\gamma \simeq \frac{43}{18}$ and give a susceptibility ratio of 200, in excellent agreement (though it may be accidental due to large distances from p_c) with the susceptibility ratio of random percolation.

We also investigated at the threshold the variation with system size of the second moment and of the strength of the largest cluster. These two quantities should vary for $L \times L$ systems as $L^{-\beta/\nu}$ and $L^{\gamma/\nu}$. We cannot reliably determine $\beta/\nu \sim 0.15$, but $\gamma/\nu \simeq 1.72$ agrees reasonably with $\frac{43}{24} = 1.79$ as predicted for random percolation. Thus again universality is confirmed; but with respect to exponents that conclusion was already drawn by Stoll and Domb (1979).

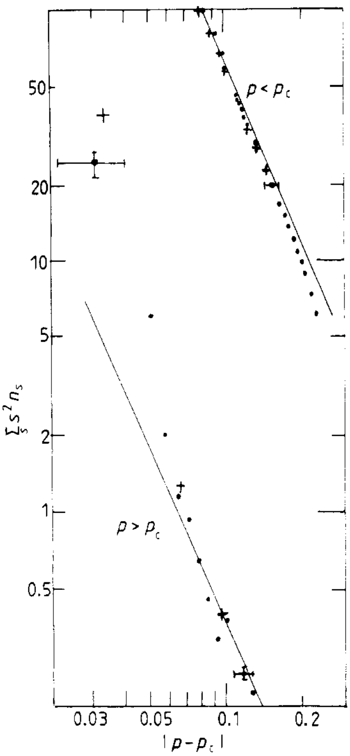

Figure 2. Log–log plot of the second moment of the cluster size distribution versus distance from the effective critical point $p_c = 0.555$. The dots refer to 800×800, the crosses to 400×400 lattices. The two parallel straight lines have the theoretically predicted slope $\frac{43}{18}$; their distance corresponds to a susceptibility ratio of 200. Note the large distance from the percolation threshold needed to get reliable data.

In summary, our results confirm that the cluster number ratio and the susceptibility ratio for this correlated percolation problem have the same 'universal' values as random percolation, in agreement with earlier predictions from the renormalisation group. We have also found that rather strong size effects are responsible for the too small value found by Stoll and Domb for the cluster number ratio.

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